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Cosmic Microwave Background:

The universe is homogeneous and isotropic at large scales, but lumpy at smaller scales (galaxies and clusters). Therefore, FRW metric describes it well at large distances and also early times. However, the universe is not an exact FRW universe. Such a universe would always be homogeneous and isotropic, in which case no structure could form.

The question of structure formation is one of the most important questions in cosmology.

The answer to this question involves the evolution of a universe with initially small inhomogeneities into a universe where inhomogeneities are very large at small scales.

The evidence that inhomogeneities were small initially comes from the observation of temperature anisotropy in the Cosmic Microwave Background (CMB).

CMB photons (arriving from the recombination epoch, $t_{rec} \sim 400{,}000$ yr) have a black-body spectrum with temperature $T \simeq 2.7°K$, and a tiny anisotropy $\frac{\delta T}{T} \sim O(10^{-5})$.

This can be clearly seen by finding the two-point correlation function of temperature anisotropy $\frac{\delta T}{T}(\vec{r})$ between points on the surface of last scattering.

Expanding this correlation function in spherical harmonic's (which makes sense because the surface of last scattering is, almost, a sphere), we can find the power spectrum.

Power spectrum at low multipoles $\ell < O(10)$ gives us information about the primordial inhomogeneities. Low ℓ corresponds to large azimuth angle θ ($\theta \propto \ell^{-1}$). The shape of power spectrum, the explanation for its, and what can be extracted about the content of the universe from that will be the subject of our discussion in the next few lectures.

There are two important facts about the power spectrum at large θ:

1- $\frac{\delta T}{T}$ is small at the time of recombination. Thus primordial inhomogeneities are small in size.

2- The power spectrum is almost flat as a function of ℓ at low ℓ. The spectrum is almost scale invariant then.

This raises two questions;

(a) How inhomogeneities with the observed features were produced?

(b) How did they evolve over time? Eventually, how are they related to the observed structure?

Generation of Density Perturbations:

First let us see how the primordial perturbations might havebeen produced. Here we discuss two general possibilities.

(1) Thermal fluctuations. The early Universe is a thermal bath of elementary particles. Inevitably, there are thermal fluctuations in quantities like number density and energy density of particles in thermal equilibrium. One possibility could be that these thermal fluctuations were the source for primordial inhomogeneities.

There are, however, two problems here:

- Thermal fluctuations result in waves that oscillate. In a static background the oscillations go on forever, if there is no damping. However, in an expanding Universe, the Hubble expansion itself introduces a damping. This implies that the amplitude of thermal fluctuations will decrease all the way until recombination. Typical initial values for the amplitude of thermal fluctuations will be redshifted by a huge factor that the corresponding inhomogeneities will be $\ll 10^{-5}$. Therefore thermal fluctuations cannot explain the observed temperature anisotropy in the CMB.

- Thermal fluctuation are at subhorizon scales. They arise as a result of causal physics, and hence their wavelength is smaller than the horizon radius at the time of creation. In a radiation dominated universe, the physical wavelength grows more slowly than the horizon radius ($\propto t^{\frac{1}{2}}$ vs $\propto t$). Therefore, they will remain within the horizon radius at all times.

What we observe at the time of recombination for low multipoles (inhomogeneities with the longest wavelength) is at a length scale ~ 30 times the horizon radius back then. These inhomogeneities could not have been due to thermal fluctuations then.

(2) Statistical fluctuations. Lets consider causally disconnected regions with a comoving radius ~ 30 times smaller than the observable size of the universe at the present time (as just mentioned). Each region contains $\sim 10^{84}$ photons on average. Now lets assume a purely random distribution of photons in the universe among these regions. The probability for a single photon to be in one of the regions is $\sim \left(\frac{1}{30}\right)^3$, which is quite small. One can therefore expects Poisson statistics, which results in $\frac{\delta n_\gamma}{n_\gamma} \sim 10^{-42}$ for a typical region. This results in a density perturbation of $O(10^{-42})$ at the time of generation.

These perturbations evolve according to general relativity. It turns out that their amplitude grows by such a huge factor between the time of production and the recombination time that the resulting $\frac{\delta T}{T}$ will be $\gg 10^{-5}$. Hence, these superhorizon statistical perturbations cannot be the source of primordial inhomogeneities either.

Now that we have entertained two general possibilities, lets get back to the question of how primordial nhomogeneities could have been created.

The situation actually is more difficult in the content of inflationary theory. Inflation is the dominant paradigm of the early universe cosmology.

It represents a brief period of superluminal expansion of the universe (meaning the physical distance between two points grows faster than the horizon radius).

In the simplest case, the scale factor "a" goes as $a \simeq a_0 \exp(Ht)$ during inflation, where $H \simeq$ Const. Such a rapid expansion erases any pre-existing inhomogeneity by blowing up the charachteristic wavelength. This implies that the universe at the end of inflation (during which a grows by a factor of $\sim 10^{60}$) is extremely homogeneous and cold.

Now the question is how primordial perturbations could have survived inflation. Actually, as it turns out, inflation also generates fluctuations itself. These are quantum fluctuations that have very tiny wavelengths, which are blown away to distances of cosmological relevance due to very rapid expansion, and amplified because of the expansion. In this context, quantum fluctuations are the source of inhomogeneities that act as the seeds for formation of cosmological structure (like galaxies and clusters). This is the best example and deepest connection between inner space and outer space.

Lets see how inflation can do this. We keep the discussion brief and focus on qualitative aspects. The exponential expansion during inflation is driven by a constant energy density associated with a Bose-Einstein condensate of a scalar field called "inflaton". The equation of motion of the zero momentum mode of the inflaton ϕ_0 is the familiar Klein-Gordon equation with a damping term related to the expansion:

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + m^2\phi_0 = 0 \qquad {}^{\cdot} \equiv \frac{d}{dt} \qquad (\text{"}m\text{" is the mass of the field})$$

Making a Fourier expansion:

$$\phi = \phi_0 + \frac{1}{(2\pi)^3}\int\left[\phi_k\, e^{-i\vec{k}\cdot\vec{r}} + \phi_k^{*}\, e^{i\vec{k}\cdot\vec{r}}\right] d^3k \,\frac{1}{\sqrt{2}(a)^{3/2}\left[\left(\frac{k}{a}\right)^2 + m^2\right]^{1/2}}$$

Here $\frac{k}{a}$ is the physical momentum of a Fourier mode and "a" is the scale factor of the universe. The equation of motion for a given mode is:

$$\ddot{\phi}_k + 3H\dot{\phi}_k + \left[\left(\frac{k}{a}\right)^2 + m^2\right]\phi_k = 0$$

Making a conformal transformation:

$$d\eta = \frac{dt}{a}, \quad \chi_k = \frac{\phi_k}{(a)^{3/2}}$$

We find:

$$\boxed{\chi_k'' + \omega_k^2\chi_k = 0 \qquad {}' \equiv \frac{d}{d\eta}, \quad \omega_k^2 = \left(\frac{k}{a}\right)^2 + m^2 - \frac{9}{4}H^2}$$

During inflation $m \ll H$.

The equation of motion of the zero momentum mode and the non-zero modes are just that of a harmonic oscillator. During inflation the zero mode is excited, while the non-zero modes are in their ground state.

Note, however, that ω_k is a function of time since $\frac{k}{a}$ is exponentially redshifted. A harmonic oscillator that is in the ground state but has a time-dependent frequency will remain in the ground state so long as the time variation in its frequency is adiabatic. Quantitatively it is defined as;

$$\omega'_k \lesssim \omega_k^2$$

Once this condition is violated, the corresponding harmonic oscillator gets excited. In the language of quantum field theory an excited oscillator is equivalent to particle production.

For a given mode "k", we have $\omega_k' \lesssim \omega_k^2$ so long as $\frac{k}{a} \ll H$. The evolution is therefore adiabatic until $\frac{k}{a} \sim H$. At this time adiabaticity is violated, and hence particles (quanta) with a physical momentum $\frac{k}{a}$ are produced. Rapid expansion does two things; it produced particle with non-zero momentum (thus inhomogeneous modes), and stretches their physical wavelength to superhorizon distances.

It turns out that the occupation number of produced particles also increases exponentially. They can be treated classically (just as a classical electromagnetic field is superposition of a large number of photons in the same state). As a result, inflation converts quantum fluctuations with very short wavelengths to physical excitations (inhomogeneities) with wavelengths of cosmological relevance.

A remarkable feature is that if $H \simeq$ const., then the inhomogeneities have an almost scale invariant spectrum, as indicated by observation. The fact that inflation can generate almost scale invariant perturbations is considered the strongest observational support for inflation.

To date inflation is the best solution to the aesthetic problems of big bang cosmology (flatness, horizon, ... problems) and generating the seeds for the structure formation. It is therefore the dominant paradigm of early universe cosmology.